Identities & Inverses

Identity - Something that doesn't change a quantity under a particular operation.

Addition: 0 $[10 + 0 = 10]$

Multiplication: 1 $[7 \cdot 1 = 7]$

Inverse - Creates the identity.

Addition: Opposites $[8 + (-8) = 0]$

Multiplication: Reciprocals $[2 \cdot \frac{1}{2} = 1]$

Why important? - Used to solve equations.

$$x + 8 = 10$$
$$-8 \quad -8$$
$$x + 0 = 2$$
$$x = 2$$

$$\frac{3}{2} \cdot \frac{2}{3} x = 10 \cdot \frac{3}{2}$$
$$1x = 15$$
$$x = 15$$

The $n \times n$ **identity matrix** is a matrix with 1's on the main diagonal and 0's elsewhere. If A is any $n \times n$ matrix and I is the $n \times n$ identity matrix, then $AI = A$ and $IA = A$.

2 × 2 Identity Matrix

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

3 × 3 Identity Matrix

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Two $n \times n$ matrices A and B are **inverses** of each other if their product (in both orders) is the $n \times n$ identity matrix. That is, $AB = I$ and $BA = I$. An $n \times n$ matrix A has an inverse if and only if $\det A \neq 0$. The symbol for the inverse of A is A^{-1}.

The Inverse of a 2 × 2 Matrix

The inverse of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

means inverse

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - cb}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \text{ provided } ad - cb \neq 0.$$

Identities & Inverses

$|A|$ = Determinant of matrix A.

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $|A| = ad - bc$

see p. 44 for Inverse formula

The inverse of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \boxed{\frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}} = \frac{1}{ad - cb}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Find $|A|$ $\begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$ $|A| = 3 \cdot 5 - (-2)(-7) = 1$

Find $A^{-1} = \frac{1}{1}\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix} = 1\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$

Verify: $\begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix} \cdot \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Find Inverse:

$A = \begin{bmatrix} 6 & 2 \\ 5 & 2 \end{bmatrix}$ $|A| = ad - bc = 12 - 10 = 2$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 2 & -2 \\ -5 & 6 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -\frac{5}{2} & 3 \end{bmatrix}$$

Verify: $\begin{bmatrix} 6 & 2 \\ 5 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 \\ -\frac{5}{2} & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$